

A VISCOSITY FORMULA FOR BINARY MIXTURES, THE  
ASSOCIATION DEGREES OF CONSTITUENTS BEING  
TAKEN INTO CONSIDERATION. VII.

By Tetsuya ISHIKAWA.

Received June 23rd, 1933. Published September 28th, 1933.

In the earlier communication of the study on this subject<sup>(1)</sup> the writer has shown that the viscosity of binary mixtures showing a maximum against the concentration, such as systems  $\text{CH}_3\text{COOH-H}_2\text{O}$ ,  $\text{CH}_3\text{OH-H}_2\text{O}$ , and  $\text{C}_2\text{H}_5\text{OH-H}_2\text{O}$ , can be explained with quite satisfaction by postulating the difference ( $\delta$ ) of measured viscosity ( $\eta$ ) from that calculated from his viscosity formula for chemically indifferent mixtures ( $\eta_0$ ) or "the solva-

---

(1) This Bulletin 4 (1929), 25.

tion viscosity" to be proportionate to the encounter probability of  $\nu_1$  and  $\nu_2$  molecules of the components to form a molecular compound. Thus,

$$\begin{aligned}\eta &= \eta_0 + \delta, \\ \eta_0 &= \eta_1 + (\eta_2 - \eta_1) \frac{k_2 a_2 z_m}{k_1 a_1 (1 - z_m) + k_2 a_2 z_m}, \\ \delta &= C(1 - z_m)^{\nu_1} z_m^{\nu_2},\end{aligned}$$

where  $\eta$ ,  $a$ ,  $k$  with suffixes 1 and 2 signify the viscosity coefficients, the association degrees and field constants of components 1 and 2 respectively;  $z_m$  a formal molar fraction of component 2; and  $C$  a proportional constant.

According to this theory the criterion for determining the composition of a molecular compound is not the position of the maximum often shifting with temperature which has given annoyance to many earlier investigators, but the keeping constancy of the proportional constant  $C$  which strictly holds good for viscosity isotherms so long as the molecular compound exists in the mixture.

#### Dependence of the Field Constant $k$ on Temperature.

Before entering into further study on the proportional constant  $C$ , it is necessary to test the dependence of the field constant on temperature. Since association degrees of liquids are functions of temperature, the mixtures which are to be taken for test of the proposition are those which must be composed of two non-associated and chemically indifferent liquids. The following case studied by Meyer and Mylius<sup>(2)</sup> is very favorable for this purpose.

The fact that is readily understood from Table 1 is that the field constant of any liquid is independent of temperature or otherwise it has the same temperature variation, and that the factor in  $\frac{k_2 a_2}{k_1 a_1}$  which changes with temperature is the ratio of the association degree of component 2 to that of component 1. So that, for such pairs as are composed of liquids having the same temperature coefficient of the association degrees, the term  $\frac{k_2 a_2}{k_1 a_1}$  is of course independent of temperature and the above equation which originally holds true for viscosity isotherms is also valid for any temperature.

---

(2) J. Meyer & B. Mylius, *Z. physik. Chem.* **95** (1920), 349.

Table 1.  
C<sub>6</sub>H<sub>6</sub>-C<sub>6</sub>H<sub>5</sub>Br, Meyer & Mylius.

$z$ -C <sub>6</sub> H <sub>5</sub> Br	0°C.	10°C.	20°C.	30°C.	40°C.	50°C.	60°C.	70°C.
0.00000 $\eta$	0.00912	0.00758	0.00652	0.00564	0.00503	0.00443	0.00392	0.00358
$\eta$	0.00976	0.00827	0.00703	0.00616	0.00546	0.00487	0.00438	0.00398
0.24746 $\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	0.37	0.46	0.38	0.43	0.39	0.44	0.50	0.39
$\eta$	0.01115	0.00926	0.00788	0.00635	0.00603	0.00567	0.00500	0.00448
0.50238 $\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	0.49	0.47	0.41	0.41	0.37	0.52	0.48	0.44
$\eta$	0.01257	0.01059	0.00915	0.00803	0.00709	0.00636	0.00579	0.00524
0.74746 $\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	0.45	0.45	0.44	0.46	0.43	0.42	0.45	0.44
1.00000 $\eta$	0.01515	0.01282	0.01117	0.00979	0.00872	0.00794	0.00719	0.00652
Mean $\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	0.44	0.46	0.41	0.43	0.40	0.46	0.48	0.42

Table 2.  
CH<sub>3</sub>OH-C<sub>2</sub>H<sub>5</sub>OH, Bingham, White, Thomas, & Cadwell.

$z$ -C <sub>2</sub> H <sub>5</sub> OH	25°C.		35°C.		45°C.		55°C.	
	$\eta$	$\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	$\eta$	$\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	$\eta$	$\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$	$\eta$	$\left(\frac{k_2 a_2}{k_1 a_1}\right)_z$
0.0000	0.00548		0.00476		0.00420		0.00371	
0.2616	0.00639	0.55	0.00552	0.59	0.00481	0.60	0.00423	0.66
0.4960	0.00737	0.53	0.00635	0.58	0.00548	0.60	0.00476	0.63
0.7385	0.00880	0.54	0.00746	0.56	0.00636	0.59	0.00546	0.62
1.0000	0.01099		0.00917		0.00766		0.00645	
	Mean	0.54		0.58		0.60		0.63

Table 3.

$$\text{H}_2\text{O}-\text{CH}_3\text{OH}, \text{Traube.}^{(3)} \quad \eta_0 = \eta_1 + (\eta_2 - \eta_1) \frac{0.96 z_m}{(1 - z_m) + 0.96 z_m}.$$

$z-\text{CH}_3\text{OH}$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00
$z_m-\text{CH}_3\text{OH}$	0.0000	0.0588	0.1233	0.1943	0.2727	0.3601	0.4576	0.6925	1.0000
20°C.	$\eta$	0.01035	0.01338	0.01639	0.01829	0.01881	0.01816	0.01629	0.01142
	$\eta_0$	—	0.01011	0.00984	0.00955	0.00922	0.00885	0.00844	0.00742
	$\delta$		0.00327	0.00655	0.00874	0.00959	0.00931	0.00785	0.00400
	$\bar{\delta}$		0.063	0.069	0.069	0.067	0.063	0.058	0.063
	$(1-z_m)^2 z_m$								
				Mean	0.065				
30°C.	$\eta$	0.00819	0.01056	0.01240	0.01378	0.01417	0.01378	0.01273	0.00933
	$\eta_0$	—	0.00803	0.00786	0.00767	0.00745	0.00722	0.00695	0.00629
	$\delta$		0.00253	0.00454	0.00611	0.00672	0.00656	0.00578	0.00304
	$\bar{\delta}$		0.049	0.048	0.048	0.047	0.045	0.043	0.046
	$(1-z_m)^2 z_m$								
				Mean	0.047				
40°C.	$\eta$	0.00668	0.00823	0.00960	0.01064	0.01091	0.01073	0.01004	0.00769
	$\eta_0$	—	0.00656	0.00644	0.00629	0.00614	0.00596	0.00576	0.00528
	$\delta$		0.00167	0.00316	0.00435	0.00477	0.00477	0.00428	0.00241
	$\bar{\delta}$		0.032	0.033	0.035	0.033	0.032	0.032	(0.037)
	$(1-z_m)^2 z_m$								
				Mean	0.033				
50°C.	$\eta$	0.00566	0.00688	0.00788	0.00867	0.00884	0.00875	0.00817	0.00650
	$\eta_0$	—	0.00557	0.00547	0.00536	0.00524	0.00510	0.00494	0.00457
	$\delta$		0.00131	0.00241	0.00331	0.00360	0.00365	0.00323	0.00193
	$\bar{\delta}$		0.025	0.025	0.026	0.025	0.025	0.024	(0.030)
	$(1-z_m)^2 z_m$								
				Mean	0.025				
60°C.	$\eta$	0.00495	0.00565	0.00645	0.00699	0.00717	0.00730	0.00685	0.00569
	$\eta_0$	—	0.00487	0.00479	0.00470	0.00460	0.00448	0.00435	0.00403
	$\delta$		0.00078	0.00166	0.00229	0.00257	0.00282	0.00250	0.00166
	$\bar{\delta}$		(0.015)	0.018	0.018	0.017	0.019	0.019	(0.025)
	$(1-z_m)^2 z_m$								
				Mean	0.018				

(3) J. Traube, *Ber.* **19** (1886), 871.

Table 4.

H<sub>2</sub>O—C<sub>2</sub>H<sub>5</sub>OH, International Critical Tables, V, 22.

$$\eta_0 = \eta_1 + (\eta_2 - \eta_1) \frac{0.70 z_m}{(1 - z_m) + 0.70 z_m}.$$

$z$ -C <sub>2</sub> H <sub>5</sub> OH	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$z_m$ -C <sub>2</sub> H <sub>5</sub> OH	0.0000	0.0417	0.0891	0.1435	0.2068	0.2811	0.3698	0.4773	0.6101	0.7787	1.0000
$\eta$	0.01794	0.03215	0.05275	0.06900	0.07150	0.06625	0.05715	0.04720	0.03648	0.02694	0.01776
$\eta_0$	—	0.01793	0.01793	0.01792	0.01791	0.01790	0.01789	0.01787	0.01785	0.01781	—
$\delta$		0.01422	0.03482	0.05108	0.05359	0.04835	0.03926	0.02933	0.01863	0.00913	
0°C. $\delta$		(0.388)	0.518	0.565	0.519	0.463	0.424	0.430	0.515	(1.08)	
$\frac{\delta}{(1-z_m)^3 z_m}$				Mean	0.491						
$\delta$							(0.267)	0.220	0.201	0.239	
$\frac{\delta}{(1-z_m)^2 z_m}$							Mean	0.232			
$\eta$	0.01310	0.02162	0.03235	0.04095	0.04355	0.04174	0.03787	0.03268	0.02663	0.02048	0.01480
$\eta_0$	—	0.01315	0.01321	0.01328	0.01336	0.01346	0.01359	0.01376	0.01399	0.01431	—
$\delta$		0.00847	0.01914	0.02767	0.03019	0.02828	0.02428	0.01892	0.01264	0.00617	
10°C. $\delta$		(0.231)	0.284	0.307	0.292	0.271	0.262	0.278	(0.350)	(0.731)	
$\frac{\delta}{(1-z_m)^3 z_m}$				Mean	0.282						
$\delta$							(0.165)	0.145	0.136	0.161	
$\frac{\delta}{(1-z_m)^2 z_m}$							Mean	0.147			
$\eta$	0.01009	0.01548	0.02168	0.02670	0.02867	0.02832	0.02642	0.02369	0.01998	0.01601	0.01221
$\eta_0$	—	0.01015	0.01023	0.01031	0.01042	0.01055	0.01071	0.01092	0.01120	0.01160	—
$\delta$		0.00533	0.01145	0.01639	0.01825	0.01777	0.01571	0.01277	0.00878	0.00441	
20°C. $\delta$		(0.145)	0.170	0.182	0.177	0.170	0.170	(0.187)	(0.243)	(0.522)	
$\frac{\delta}{(1-z_m)^3 z_m}$				Mean	0.174						
$\delta$							0.107	0.098	0.095	0.116	
$\frac{\delta}{(1-z_m)^2 z_m}$							Mean	0.104			
$\eta$	0.00800	0.01153	0.01539	0.01849	0.01991	0.02001	0.01906	0.01744	0.01519	0.01270	0.00997
$\eta_0$	—	0.00806	0.00813	0.00821	0.00830	0.00842	0.00857	0.00877	0.00903	0.00940	—
$\delta$		0.00347	0.00726	0.01028	0.01161	0.01159	0.01049	0.00867	0.00616	0.00330	
30°C. $\delta$		(0.095)	0.108	0.114	0.113	0.111	0.113	(0.127)	(0.170)	(0.391)	
$\frac{\delta}{(1-z_m)^3 z_m}$				Mean	0.112						
$\delta$							0.071	0.066	0.066	(0.086)	
$\frac{\delta}{(1-z_m)^2 z_m}$							Mean	0.068			

Table 4.—(Concluded)

$z$ -C <sub>2</sub> H <sub>5</sub> OH	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$z_m$ -C <sub>2</sub> H <sub>5</sub> OH	0.0000	0.0417	0.0891	0.1435	0.2068	0.2811	0.3698	0.4773	0.6101	0.7787	1.0000
$\eta$	0.00654	0.00896	0.01144	0.01353	0.01455	0.01475	0.01426	0.01328	0.01181	0.01022	0.00824
$\eta_0$	—	0.00659	0.00665	0.00672	0.00680	0.00691	0.00704	0.00720	0.00743	0.00775	—
$\delta$		0.00237	0.00479	0.00681	0.00775	0.00784	0.00722	0.00608	0.00438	0.00247	
40°C. $\frac{\delta}{(1-z_m)^3 z_m}$		(0.065)	0.071	0.075	0.075	0.075	(0.078)	(0.089)	(0.121)	(0.293)	
$\delta$				Mean	0.074						
$\frac{\delta}{(1-z_m)^2 z_m}$						(0.054)	0.049	0.047	0.047	(0.065)	
							Mean	0.048			
$\eta$	0.00549	0.00725	0.00896	0.01038	0.01116	0.01136	0.01109	0.01044	0.00950	0.00835	0.00695
$\eta_0$	—	0.00553	0.00558	0.00565	0.00572	0.00580	0.00592	0.00606	0.00625	0.00653	—
$\delta$		0.00172	0.00338	0.00473	0.00544	0.00556	0.00517	0.00438	0.00325	0.00182	
50°C. $\frac{\delta}{(1-z_m)^3 z_m}$		(0.047)	0.050	0.052	0.053	0.053	(0.056)	(0.064)	(0.090)	(0.216)	
$\delta$				Mean	0.052						
$\frac{\delta}{(1-z_m)^2 z_m}$						(0.038)	0.035	0.034	0.035	(0.048)	
							Mean	0.035			
$\eta$	0.00470	0.00602	0.00728	0.00826	0.00887	0.00904	0.00887	0.00841	0.00778	0.00695	0.00590
$\eta_0$	—	0.00474	0.00478	0.00483	0.00489	0.00496	0.00505	0.00517	0.00533	0.00555	—
$\delta$		0.00128	0.00250	0.00343	0.00398	0.00408	0.00382	0.00324	0.00245	0.00140	
60°C. $\frac{\delta}{(1-z_m)^3 z_m}$		(0.035)	0.037	0.038	0.039	0.039	(0.041)	(0.047)	(0.068)	(0.166)	
$\delta$				Mean	0.038						
$\frac{\delta}{(1-z_m)^2 z_m}$						(0.028)	0.026	0.025	0.026	(0.039)	
							Mean	0.026			
$\eta$	0.00407	0.00509	0.00606	0.00677	0.00724	0.00739	0.00727	0.00696	0.00648	0.00549	0.00506
$\eta_0$	—	0.00410	0.00413	0.00417	0.00422	0.00428	0.00436	0.00446	0.00459	0.00477	—
$\delta$		0.00099	0.00193	0.00260	0.00302	0.00311	0.00291	0.00250	0.00189	0.00072	
70°C. $\frac{\delta}{(1-z_m)^3 z_m}$		(0.027)	0.029	0.029	0.029	0.030	(0.031)	(0.037)	(0.052)	(0.085)	
$\delta$				Mean	0.029						
$\frac{\delta}{(1-z_m)^2 z_m}$						(0.021)	0.020	0.019	0.020	0.019	
							Mean	0.020			

Table 5.

H<sub>2</sub>O—CH<sub>3</sub>COOH, International Critical Tables, V, 20.

$$\eta_0 = \eta_1 + (\eta_2 - \eta_1) \frac{1.29 z_m}{(1 - z_m) + 1.29 z_m}.$$

$z$ —CH <sub>3</sub> COOH	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$z_m$ —CH <sub>3</sub> COOH	0.0000	0.0323	0.0658	0.1140	0.1677	0.2417	0.3104	0.4120	0.4737	0.7300	1.0000
$\eta$	0.01145	0.01360	0.01630	0.01910	0.02160	0.02475	0.02765	0.03020	0.03105	0.02680	0.01350
$\eta_0$	—	0.01153	0.01163	0.01174	0.01187	0.01205	0.01220	0.01242	0.01255	0.01304	—
15°C. $\delta$		0.00207	0.00467	0.00736	0.00973	0.01270	0.01545	0.01778	0.01850	0.01376	
$\delta$		(0.066)	0.072	0.073	0.070	0.069	0.072	0.073	0.074	0.070	
$(1-z_m)z_m$					Mean	0.072					
$\eta$	0.01009	0.01210	0.01415	0.01635	0.01870	0.02135	0.02390	0.02640	0.02695	0.02310	0.01265
$\eta_0$	—	0.01020	0.01032	0.01045	0.01062	0.01084	0.01103	0.01131	0.01147	0.01208	—
20°C. $\delta$		0.00190	0.00383	0.00590	0.00808	0.01051	0.01287	0.01509	0.01548	0.01102	
$\delta$		0.061	0.059	0.058	0.058	0.057	0.060	0.062	0.062	(0.056)	
$(1-z_m)z_m$					Mean	0.060					
$\eta$	0.00895	0.01065	0.01250	0.01450	0.01685	0.01870	0.02085	0.02290	0.02365	0.02050	0.01155
$\eta_0$	—	0.00906	0.00918	0.00932	0.00949	0.00971	0.00991	0.01019	0.01035	0.01097	—
25°C. $\delta$		0.00159	0.00332	0.00518	0.00706	0.00899	0.01094	0.01271	0.01330	0.00953	
$\delta$		0.051	0.051	0.051	0.051	0.049	0.051	0.053	0.053	0.048	
$(1-z_m)z_m$					Mean	0.051					
$\eta$	0.00800	0.00955	0.01110	0.01290	0.01475	0.01660	0.01850	0.02040	0.02095	0.01840	0.01065
$\eta_0$	—	0.00811	0.00823	0.00838	0.00855	0.00877	0.00897	0.00926	0.00942	0.01006	—
30°C. $\delta$		0.00144	0.00287	0.00452	0.00620	0.00783	0.00953	0.01114	0.01153	0.00834	
$\delta$		0.046	0.044	0.045	0.044	0.043	0.045	0.046	0.046	0.042	
$(1-z_m)z_m$					Mean	0.045					
$\eta$	0.00721	0.00855	0.01000	0.01150	0.01315	0.01480	0.01650	0.01815	0.01855	0.01655	0.00990
$\eta_0$	—	0.00732	0.00745	0.00759	0.00776	0.00799	0.00820	0.01849	0.00866	0.00930	—
35°C. $\delta$		0.00123	0.00255	0.00391	0.00539	0.00681	0.00830	0.00966	0.00989	0.00725	
$\delta$		0.039	0.039	0.039	0.039	0.037	0.039	0.040	0.040	0.037	
$(1-z_m)z_m$					Mean	0.039					

Table 5.—(Continued)

$z-\text{CH}_3\text{COOH}$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$z_m-\text{CH}_3\text{COOH}$	0.0000	0.0323	0.0698	0.1140	0.1677	0.2417	0.3104	0.4120	0.4737	0.7300	1.0000
$\eta$	0.00654	0.00780	0.00905	0.01040	0.01180	0.01325	0.01470	0.01620	0.01660	0.01490	0.00925
$\eta_0$	—	0.00665	0.00678	0.00692	0.00709	0.00733	0.00753	0.00782	0.00799	0.00864	—
40°C. $\delta$	—	0.00115	0.00227	0.00348	0.00471	0.00592	0.00717	0.00838	0.00861	0.00626	—
$\delta$	—	(0.037)	0.035	0.035	0.034	0.032	0.034	0.035	0.035	0.032	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.034	—	—	—	—	—
$\eta$	0.00597	0.00710	0.00820	0.00940	0.01060	0.01185	0.01320	0.01445	0.01485	0.01345	0.00865
$\eta_0$	—	0.00608	0.00620	0.00635	0.00652	0.00674	0.00694	0.00723	0.00739	0.00803	—
45°C. $\delta$	—	0.00102	0.00200	0.00305	0.00408	0.00511	0.00626	0.00722	0.00746	0.00542	—
$\delta$	—	(0.033)	0.031	0.030	0.029	0.028	0.029	0.030	0.030	0.028	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.029	—	—	—	—	—
$\eta$	0.00549	0.00650	0.00745	0.00855	0.00965	0.01080	0.01190	0.01305	0.01340	0.01225	0.00810
$\eta_0$	—	0.00650	0.00572	0.00586	0.00603	0.00625	0.00645	0.00673	0.00689	0.00752	—
50°C. $\delta$	—	0.00090	0.00173	0.00269	0.00362	0.00455	0.00545	0.00632	0.00651	0.00478	—
$\delta$	—	(0.029)	0.027	0.027	0.026	0.025	0.026	0.026	0.026	0.024	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.026	—	—	—	—	—
$\eta$	0.00507	0.00600	0.00680	0.00775	0.00880	0.00985	0.01080	0.01195	0.01215	0.01125	0.00760
$\eta_0$	—	0.00517	0.00529	0.00543	0.00559	0.00581	0.00600	0.00627	0.00643	0.00704	—
55°C. $\delta$	—	0.00083	0.00151	0.00232	0.00321	0.00404	0.00480	0.00568	0.00572	0.00421	—
$\delta$	—	(0.027)	0.023	0.023	0.023	0.022	0.022	0.023	0.023	0.021	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.023	—	—	—	—	—
$\eta$	0.00470	0.00550	0.00625	0.00710	0.00800	0.00900	0.00995	0.01090	0.01105	0.01030	0.00700
$\eta_0$	—	0.00479	0.00490	0.00503	0.00517	0.00537	0.00555	0.00579	0.00594	0.00649	—
60°C. $\delta$	—	0.00071	0.00135	0.00207	0.00283	0.00363	0.00440	0.00511	0.00511	0.00381	—
$\delta$	—	(0.023)	0.021	0.021	0.020	0.020	0.021	0.021	0.021	0.019	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.020	—	—	—	—	—
$\eta$	0.00437	0.00505	0.00575	0.00655	0.00735	0.00820	0.00900	—	0.01010	—	0.00675
$\eta_0$	—	0.00447	0.00458	0.00471	0.00486	0.00506	0.00524	—	0.00565	—	—
65°C. $\delta$	—	0.00058	0.00117	0.00184	0.00249	0.00314	0.00376	—	0.00445	—	—
$\delta$	—	(0.019)	0.018	0.018	0.018	0.017	0.018	—	0.018	—	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.018	—	—	—	—	—



Table 5.—(Concluded)

$z-\text{CH}_3\text{COOH}$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$z_m-\text{CH}_3\text{COOH}$	0.0000	0.0323	0.0698	0.1140	0.1677	0.2417	0.3104	0.4120	0.4737	0.7300	1.0000
$\eta$	0.00381	0.00440	0.00495	0.00560	0.00630	0.00700	0.00765	—	0.00855	—	0.00605
$\eta_0$	—	0.00390	0.00401	0.00413	0.00427	0.00446	0.00463	—	0.00501	—	—
75°C. $\delta$	—	0.00050	0.00094	0.00147	0.00203	0.00254	0.00302	—	0.00354	—	—
$\delta$	—	0.016	0.015	0.015	0.015	0.014	0.014	—	0.014	—	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.015	—	—	—	—	—
$\eta$	0.00336	0.00385	0.00435	0.00490	0.00545	0.00605	0.00660	—	0.00740	—	0.00545
$\eta_0$	—	0.00345	0.00354	0.00366	0.00379	0.00397	0.00413	—	0.00448	—	—
85°C. $\delta$	—	0.00040	0.00081	0.00124	0.00166	0.00208	0.00247	—	0.00292	—	—
$\delta$	—	0.013	0.013	0.012	0.012	0.011	0.012	—	0.012	—	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.012	—	—	—	—	—
$\eta$	0.00299	0.00340	0.00385	0.00430	0.00475	0.00530	0.00575	—	0.00640	—	0.00490
$\eta_0$	—	0.00307	0.00316	0.00326	0.00338	0.00355	0.00369	—	0.00402	—	—
95°C. $\delta$	—	0.00033	0.00069	0.00104	0.00137	0.00175	0.00206	—	0.00238	—	—
$\delta$	—	0.011	0.011	0.010	0.010	0.010	0.010	—	0.010	—	—
$(1-z_m)z_m$	—	—	—	—	Mean	0.010	—	—	—	—	—

Table 6.

Molecular compound  $\text{CH}_3\text{OH} \cdot 2\text{H}_2\text{O}$ .  $\log C_{calc.} = 2.964 + \frac{2.00}{2+1} \log (\eta_1^2 \eta_2)$ .

Temp. C.	$\eta_1$	$\eta_2$	$C_{obs.}$	$C_{calc.}$	$\eta_m$
20	0.01035	0.00607	0.065	0.069	6.3
30	0.00819	0.00541	0.047	0.047	4.3
40	0.00668	0.00463	0.033	0.032	3.0
50	0.00566	0.00406	0.025	0.024	2.2
60	0.00495	0.00361	0.018	0.018	1.7

Table 7.

Molecular compound  $C_2H_5OH \cdot 3H_2O$ .  $\log C_{calc.} = 3.191 + \frac{2.00}{3+1} \log(\eta_1^3 \eta_2)$ .

Temp. C.	$\eta_1$	$\eta_2$	$C_{obs.}$	$C_{calc.}$	$\eta_m$
0	0.01794	0.01776	0.491	0.498	773
10	0.01310	0.01480	0.282	0.283	439
20	0.01009	0.01221	0.174	0.174	270
30	0.00800	0.00997	0.112	0.111	172
40	0.00654	0.00824	0.074	0.074	115
50	0.00549	0.00695	0.052	0.053	82
60	0.00470	0.00590	0.038	0.038	59
70	0.00407	0.00506	0.029	0.029	45

Table 8.

Molecular compound  $C_2H_5OH \cdot 2H_2O$ .  $\log C_{calc.} = 2.612 + \frac{1.83}{2+1} \log(\eta_1^2 \eta_2)$ .

Temp. C.	$\eta_1$	$\eta_2$	$C_{obs.}$	$C_{calc.}$	$\eta_m$
0	0.01794	0.01776	0.232	0.265	108
10	0.01310	0.01480	0.147	0.145	59
20	0.01009	0.01221	0.104	0.102	42
30	0.00800	0.00997	0.068	0.068	28
40	0.00654	0.00824	0.048	0.048	20
50	0.00549	0.00695	0.035	0.035	14
60	0.00470	0.00590	0.026	0.026	11
70	0.00407	0.00506	0.020	0.020	8.2

Table 9.

Molecular compound  $\text{CH}_3\text{COOH} \cdot \text{H}_2\text{O}$ .  $\log C_{\text{calc.}} = 2.065 + \frac{1.68}{1+1} \log(\eta_1\eta_2)$ .

Temp. C.	$\eta_1$	$\eta_2$	$C_{\text{obs.}}$	$C_{\text{calc.}}$	$\eta_m$
15	0.01145	0.01350	0.072	0.073	8.5
20	0.01009	0.01265	0.060	0.062	7.2
25	0.00895	0.01155	0.051	0.052	6.0
30	0.00800	0.01065	0.045	0.044	5.1
35	0.00721	0.00990	0.039	0.038	4.4
40	0.00654	0.00925	0.034	0.033	3.8
45	0.00597	0.00865	0.029	0.029	3.4
50	0.00549	0.00810	0.026	0.026	3.0
55	0.00507	0.00760	0.023	0.023	2.7
60	0.00470	0.00700	0.020	0.020	2.3
65	0.00437	0.00675	0.018	0.018	2.1
75	0.00381	0.00605	0.015	0.015	1.7
85	0.00336	0.00545	0.012	0.012	1.4
95	0.00299	0.00490	0.010	0.010	1.2

As to the temperature variation of association degrees Ramsay and Shields<sup>(4)</sup> calculated those of the abnormal liquids-water, methyl alcohol, ethyl alcohol and acetic acid from the deviation of their surface tension equation. Apart from the justification of their calculation for the absolute values of association degrees, we acknowledge from their results that the decreasing rates of molecular association of these liquids with rising temperature are nearly equal, temperature coefficients being  $-0.0030 \sim -0.0035$ .

A further test for methyl and ethyl alcohols with the viscosity data observed by Bingham, White, Thomas and Cadwell<sup>(5)</sup> affirms the fact.

Moreover as already studied, the solvation viscosities of systems  $\text{CH}_3\text{COOH} \cdot \text{H}_2\text{O}$ ,  $\text{CH}_3\text{OH} \cdot \text{H}_2\text{O}$  and  $\text{C}_2\text{H}_5\text{OH} \cdot \text{H}_2\text{O}$  have high values in comparison with  $\eta_0$  at moderate concentrations, it is quite sufficient to regard that in the calculation of  $\eta_0$  of each system  $\frac{k_2 a_2}{k_1 a_1}$  is independent of temperature. In the above calculation, therefore,  $\frac{k_2 a_2}{k_1 a_1}$  in each system has been put to be the same value as obtained in the previous paper.

(4) W. Ramsay & J. Shields, *Z. physik. Chem.* **12** (1893), 468; W. Ramsay, *Z. physik. Chem.* **15** (1894), 113.

(5) E. C. Bingham, G. F. White, A. Thomas, J. L. Cadwell, *Z. physik. Chem.* **83** (1913), 652.

### The Viscosity Coefficient of a Molecular Compound and its Stability Coefficient.

As the theory demands, the proportional constant  $C$  in each system has proved to keep constancy for isothermal mixtures. The next study is its temperature variation.

Since the proportional constant  $C$  must involve in it the term which corresponds to the viscosity of a molecular compound, it is expected that there may exist some relationship between  $C$  and the viscosities of components  $\eta_1$  and  $\eta_2$ , and if any, the component viscosities may probably take part in the form  $\eta_1^{\nu_1} \eta_2^{\nu_2}$  as considered from the composition term  $(1-z_m)^{\nu_1} z_m^{\nu_2}$ . Then we may put  $C$  in the following form:

$$C = C_0(\eta_1^{\nu_1} \eta_2^{\nu_2})^{\frac{m}{\nu_1 + \nu_2}},$$

where  $C_0$  and  $m$  are constants independent of temperature.

The test of the postulation with the molecular compounds in the above three pairs is extremely good. The results of the calculation are tabulated in Tables 6, 7, 8 and 9.

Transform the expression into

$$\frac{C}{C_0} = (\eta_1^{\nu_1} \eta_2^{\nu_2})^{\frac{m}{\nu_1 + \nu_2}}.$$

The left-hand side of the equation is no other than the viscosity coefficient of a molecular compound and let it be denoted by  $\eta_m$ , then

$$\eta_m = (\eta_1^{\nu_1} \eta_2^{\nu_2})^{\frac{m}{\nu_1 + \nu_2}},$$

or taking logarithms of both sides

$$\log \eta_m = m \left\{ \frac{\nu_1}{\nu_1 + \nu_2} \log \eta_1 + \frac{\nu_2}{\nu_1 + \nu_2} \log \eta_2 \right\} \dots \dots \dots (1)$$

Kendall<sup>(6)</sup> showed, with eighty four mixtures of presumably non-associated and chemically indifferent liquids, that the deviation from the straight line is generally least when the composition in the Arrhenius

---

(6) J. Kendall, Medd. K. Vetenskapsakad. *Nobelinst.* **2**, No. 25 (1913), 1.

logarithmic formula is expressed as molar fraction, i.e. in the same concentration as above

$$\log \eta = \frac{\nu_1}{\nu_1 + \nu_2} \log \eta_1 + \frac{\nu_2}{\nu_1 + \nu_2} \log \eta_2 \dots\dots\dots (2)$$

The average divergence of this formula from the observed viscosity values is estimated by him to be  $\pm 2.3\%$  or  $m = 1 \pm 0.02$ .

Comparing (1) with (2) it is understood that the greater the difference of  $m$  from unity the more stable the molecular compound may be in the mixture. The  $m$  is called hereafter "the stability coefficient" of a molecular compound. In the following table are summarized the stability coefficients of the molecular compounds above studied.

Molecular compound	$m$
$\text{CH}_3\text{OH} \cdot 2\text{H}_2\text{O}$	2.00
$\text{C}_2\text{H}_5\text{OH} \cdot 3\text{H}_2\text{O}$	2.00
$\text{C}_2\text{H}_5\text{OH} \cdot 2\text{H}_2\text{O}$	1.83
$\text{CH}_3\text{COOH} \cdot \text{H}_2\text{O}$	1.63

Among the four hydrates the hydrates  $\text{C}_2\text{H}_5\text{OH} \cdot 3\text{H}_2\text{O}$  has  $\eta_m$  of the greatest value which suggests the possibility of crystallization of this hydrate at lower temperatures. In fact, Tammann and Pillsbury<sup>(7)</sup> reported to yield transparent crystals of a hydrate by cooling the mixtures of 34–43 wt. % (20–30 mol %) alcohol which they attributed to a tetrahydrate, simply considering the admixing composition, but observed no separation of crystalline hydrates of methyl alcohol and acetic acid. The writer also observed transparent crystals of cubic system by cooling the mixtures of molar ratios of from 1  $\text{C}_2\text{H}_5\text{OH}$ : 1  $\text{H}_2\text{O}$  to 1  $\text{C}_2\text{H}_5\text{OH}$ : 4  $\text{H}_2\text{O}$  with solid carbon dioxide and alcohol freezer. From the writer's viewpoint of the present study, however, this hydrate of ethyl alcohol is undoubtedly a trihydrate.

*The Institute of Physical and Chemical Research,  
Hongo, Tokyo.*

---

(7) G. Tammann & M. E. Pillsbury, *Z. anorg. allg. Chem.* **172** (1928), 243.